# Electronic stopping power of plasma for heavy ions at low velocity

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The electronic stopping power of hot plasma for low-velocity heavy ions is studied using quantum scattering theory. The $Z_1$ oscillation of electronic stopping powers of plasma targets is more evident than that of cold solids at relatively low temperature and it becomes weaker with increasing plasma electron temperature. [S1063-651X(98)07709-5]

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The interactions between ion beams and plasmas are important for diagnosing cooling, and heating of plasma. Heavy-ion beams have been considered as drivers for inertial confinement fusion [1]. A number of experimental [2–4] and theoretical results [5–10] on the ion-beam interaction with plasma have been reported. The important conclusion drawn from these studies is that the energy losses of heavy ions penetrating hot plasmas are remarkably enhanced, as compared to the energy losses in cold solids, and the effective charges of the ions are substantially increased.

The $Z_1$ oscillation of the electronic stopping power in cold solid matter for low-velocity heavy ions is well known both from experiments [11,12] and from theoretical predictions [13,14], while in the case of hot plasmas the stopping behavior varying with $Z_1$ at low velocity has not yet been systematically studied. In this report we calculate the electronic stopping powers of hot plasma targets for heavy ions at low velocity within the framework of quantum scattering theory.

According to the scattering theory in an approximation of binary collision, we calculate the average energy loss for an arbitrary ion with velocity $\vec{v}$ moving through an electron gas. The energy loss $-dW$ over the distance $dR$ is given by

$$-dW=-d\left(\frac{\vec{p}^2}{2M_1}\right)=-\vec{v}\cdot d\vec{p}=v(-dp_\parallel), \tag{1}$$

where $-dp_\parallel$ is the momentum loss in the direction of $\vec{v}$ and $M_1$ is the mass of the ion. Then the electronic stopping power of the electron gas for the ion is given by

$$\left(-\frac{dW}{dR}\right)_e=\frac{dR}{dt}\,\frac{-dp_\parallel}{dR}=-\frac{dp_\parallel}{dt}. \tag{2}$$

As observed from the rest system of the ion, an electron with velocity $\vec{v}_e$ in the laboratory system approaches the scattering center with a velocity $\vec{w}_e=\vec{v}_e-\vec{v}$. Considering that the mass of the ion is much greater than that of an electron, after scattering through an angle $\theta$, the electron moves with a velocity $\vec{w}_e'$, where $|\vec{w}_e'|=|\vec{w}_e|$. In an electron gas, as many as $f(\vec{w}_e,\vec{v})d^3\vec{w}_e$ electrons with velocity $\vec{w}_e\sim\vec{w}+d\vec{w}$ relative to the ion scattered to a solid angle $d\Omega$ around $\theta$ will obtain a momentum transfer $-dp_\parallel(\vec{w}_e,\theta)$ from the incident ion, i.e.,

$$-dp_\parallel(\vec{w}_e,\theta)=f(\vec{w}_e,\vec{v})d^3\vec{w}_e w_e dt\ \sigma(\theta,\vec{w}_e)\times d\Omega(\vec{w}_e'-\vec{w}_e)\cdot\frac{\vec{v}}{v}m_e, \tag{3}$$

where $f(\vec{w}_e,\vec{v})$ is the distribution function of electrons, $m_e$ is the mass of an electron, and $\sigma(\theta,\vec{w}_e)$ is the differential scattering cross section. Since $w_e'=w_e$, one has

$$\vec{w}_e'-\vec{w}_e=w_e(\cos\theta)\frac{\vec{w}_e}{w_e}+w_e(\sin\theta)\frac{\vec{w}_\perp}{w_\perp}-\vec{w}_e =-(1-\cos\theta)\vec{w}_e+w_e(\sin\theta)\frac{\vec{w}_\perp}{w_\perp}, \tag{4}$$

where $\vec{w}_\perp/w_\perp$ is a unit vector perpendicular to $\vec{w}_e$. To maintain symmetry around $\vec{w}_e$, the sum of the contributions from $w_e(\sin\theta)(\vec{w}_\perp/w_\perp)\cdot(\vec{v}/v)$ must be zero. Therefore, the factor $\vec{w}_e'$ in Eq. (3) can be replaced by the first term on the right-hand side of Eq. (4). Thus we have

$$-dp_\parallel(\vec{w}_e,\theta)=-m_e f(\vec{w}_e,\vec{v})d^3\vec{w}_e w_e\sigma(\theta,\vec{w}_e)\times d\Omega(1-\cos\theta)\frac{\vec{w}_e\cdot\vec{v}}{v}dt. \tag{5}$$

Integrating over all values of $\theta$, one obtains

$$-\frac{dp_\parallel(\vec{w}_e)}{dt}=-m_e f(\vec{w}_e,\vec{v})d^3\vec{w}_e w_e\frac{\vec{w}_e\cdot\vec{v}}{v}\times\int_0^\pi(1-\cos\theta)\sigma(\theta,\vec{w}_e)2\pi\sin\theta d\theta. \tag{6}$$

The transport cross section $\sigma_{\rm tr}(\vec{w}_e)$ is defined as

$$\sigma_{\rm tr}(\vec{w}_e)=\int_0^\pi(1-\cos\theta)\sigma(\theta,\vec{w}_e)2\pi\sin\theta d\theta. \tag{7}$$

We can rewrite Eq. (6) in the form

 

$$-\frac{dp_{\parallel}(\vec{w}_e)}{dt}=-m_e f(\vec{w}_e,\vec{v})d^3\vec{w}_e w_e \sigma_{\rm tr}(\vec{w}_e)\frac{\vec{w}_e\cdot\vec{v}}{v}. \quad (8)$$

The transport cross section $\sigma_{\rm tr}(\vec{w}_e)$ is the central quantity and is determined by the scattering potential. In general, the scattering potential is complex and has no spherical symmetry. Thus the transport cross section is difficult to solve. However, in some situations when the scattering potential has spherical symmetry, the transport cross section could be solved by standard phase shifts analyses. This is the case for the penetrating ions with slow velocity [13]. Therefore, when a slow ion penetrates a hot plasma, we assume that the scattering potential between the slow ion and the scattered electrons is spherically symmetric. As a result, the transport cross section does not depend on the directions of the electron velocity $\vec{w}_e$, i.e.,

$$\sigma_{\rm tr}(\vec{w}_e)=\sigma_{\rm tr}(w_e). \quad (9)$$

In general, when an ion penetrates a hot plasma at velocity $\vec{v}$, the ion experiences the velocity distribution of the electrons in the plasma as a shifted Maxwellian distribution [8], i.e.,

$$f(\vec{w}_e,\vec{v})d^3\vec{w}_e=n_e\left(\frac{m_e}{2\pi k_B T_e}\right)^{3/2}e^{-m_e(\vec{w}_e+\vec{v})^2/2k_B T_e}d^3\vec{w}_e \quad (10)$$

where $n_e$ and $T_e$ are the plasma electron density and temperature, respectively, and $k_B$ is the Boltzmann constant. However, for $v\rightarrow 0$, as the case for slow ions penetrating a hot plasma in which we are interested in the present work, the distribution can be approximately reduced to a normal Maxwellian distribution function

$$f(v_e)d^3\vec{v}_e=n_e\left(\frac{m_e}{2\pi k_B T_e}\right)^{3/2}e^{-m_e v_e^2/2k_B T_e}d^3\vec{v}_e. \quad (11)$$

Substituting this distribution function for the one in Eq. (8) and integrating over all possible velocities of the electrons, one obtains the total momentum transfer rate from the ion to the electron gas

$$-\frac{dp_{\parallel}}{dt}=-m_e\int f(v_e)w_e\sigma_{\rm tr}(w_e)\frac{\vec{w}_e\cdot\vec{v}}{v}d^3\vec{v}_e. \quad (12)$$

The angle between $\vec{v}_e$ and $\vec{v}$ is defined as $\vartheta$ and the relation among $w_e$, $v_e$, and $v$ is

$$w_e=\sqrt{v_e^2+v^2-2v_e v\cos\vartheta}. \quad (13)$$

Considering the low-velocity limit, we can expand Eqs. (13) and (9) to the first order of the velocity of the heavy ion $v$ and obtain

$$w_e\approx v_e-v\cos\vartheta \quad (14)$$

and

$$\sigma_{\rm tr}(w_e)\approx\sigma_{\rm tr}(v_e)-\frac{d\sigma_{\rm tr}(v_e)}{dv_e}v\cos\vartheta. \quad (15)$$

Equation (12) can be approximated as

$$\left(-\frac{dW}{dR}\right)_e=m_e\int f(v_e)(v_e-v\cos\vartheta)\times\left(\sigma_{\rm tr}(v_e)-\frac{d\sigma_{\rm tr}(v_e)}{dv_e}v\cos\vartheta\right)\times(v-v_e\cos\vartheta)2\pi\sin\vartheta\, d\vartheta\, v_e^2 dv_e. \quad (16)$$

After performing the integral of $\vartheta$ from 0 to $\pi$, leaving the terms including $v$ up to first order, one gets

$$\left(-\frac{dW}{dR}\right)_e=\frac{4\pi}{3}m_e v\int v_e^3 f(v_e)\times\left(4\sigma_{\rm tr}(v_e)+v_e\frac{d\sigma_{\rm tr}(v_e)}{dv_e}\right)dv_e. \quad (17)$$

The remaining problem is the treatment of the transport cross section. First of all, one must get the scattering potential. We do this in the framework of linear response theory. When an ion with charge $Z$ is placed in a plasma, from Poisson's equation, the potential in momentum space is [15]

$$\tilde{V}(\vec{q})=-\frac{4\pi Ze^2}{q^2\epsilon(q,\Omega)}, \quad (18)$$

where $\vec{q}$ is the momentum transfer to the ion, $\Omega=\vec{q}\cdot\vec{v}$, and $\epsilon(q,\Omega)$ is the dielectric function of the plasma. It has been known that the interaction between the particle and the longitudinal field can be described by the dynamic plasma polarization [16]. Thus we use the longitudinal dielectric function of the plasma $\epsilon_l(q,\Omega)$ [17],

$$\epsilon_l(q,\Omega)=1+(q\Lambda)^{-2}[1-\varphi(z)+i\sqrt{\pi}ze^{-z^2}], \quad (19)$$

where $z=(\sqrt{2}qv_T/\Omega)^{-1}$, $v_T(=\sqrt{T_e/m_e})$ is the electron thermal velocity, and $\varphi$ is the plasma dispersion function

$$\varphi(z)=2ze^{-z^2}\int_0^z e^{y^2}dy. \quad (20)$$

Thus, through a Fourier transformation one obtains the interaction potential in real space

$$V(\vec{r})=(2\pi)^{-3}\int d^3\vec{q}\, e^{-i\vec{q}\cdot\vec{r}}\tilde{V}(\vec{q}), \quad (21)$$

where $\vec{r}$ is the position vector of the plasma electron with respect to the ion. We make a reasonable approximation that $V(\vec{r})$ is spherically symmetric for a low-velocity heavy ion penetrating the plasma. Then the transport cross section $\sigma_{\rm tr}(v_e)$ can be determined using a phase shift analysis. In terms of the scattering phase shifts $\delta_l$ we have [21]

$$\sigma_{\rm tr}(v_e)=\frac{4\pi\hbar^2}{m_e^2 v_e^2}\sum_{l=0}^{\infty}(l+1)\sin^2[\delta_l(v_e)-\delta_{l+1}(v_e)], \quad (22)$$

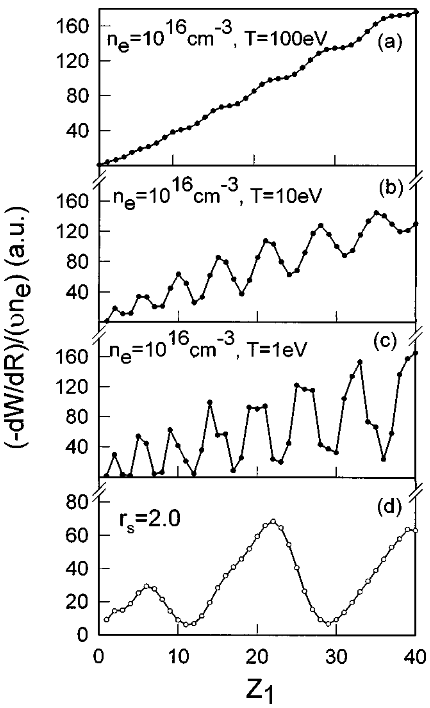


FIG. 1. $Z_1$ oscillation of the electronic stopping power of plasma targets for heavy ions at low velocity: (a)–(c) plasma stopping and (d) stopping power data for cold solid.

where $l$ is the angular momentum quantum number. The scattering phase shifts $\delta_l$ are calculated by solving the radial *Schrödinger* equation using the interaction potential given by Eq. (21).

In order to find the dependence of the electronic stopping power of the plasma on the atomic number of the penetrating heavy ion, we calculate the phase shifts for ions with atomic number $Z_1$ changing from 1 to 40. Considering the ionization and recombination processes at low velocity in the plasma, the projectile ion with atomic number $Z_1$ almost falls into its neutral charge state. The penetrating ion with its screening electron cloud can be considered a quasiatom. Then the charge distribution around the penetrating ion becomes

$$\rho(\vec{r}) = Z_1 e\,\delta(\vec{r}-\vec{v}_i t) + \rho_b(\vec{r}-\vec{v}_i t), \tag{23}$$

$$\rho_b(\vec{r}) = \sum_{\text{occ}} |\psi_b(\vec{r})|^2, \tag{24}$$

$$\int \rho_b(\vec{r})\, d^3\vec{r} = -Z_1 e, \tag{25}$$

where, as an approximation, the $\Psi_b$'s are the Roothan-Hartree-Fock wave functions for neutral ions [18].

The results shown in Figs. 1(a), 1(b), and 1(c) are the electronic stopping powers of the plasma, with electron density $n_e = 10^{16}\ \text{cm}^{-3}$ and electron temperature $T_e = 100$, 10, and 1 eV, respectively, as functions of the atomic number of the incident ion. For convenience, in the figures of this work the reduced stopping power $(dW/dR)/\upsilon n_e$ is given in atomic units, in which $m_e = \hbar = e = 1$. For the purpose of comparison, the electronic stopping power for a cold solid with density parameter $r_s = 2.0$ a.u. is shown in Fig. 1(d), which is calculated using scattering theory [19,20]. From this figure one finds that the stopping cross section of the plasma targets is larger than that of the cold solid. This so-called stopping power enhancement in hot plasmas has been reported [2–4]. The physical origination of the energy loss enhancement in hot plasmas was discussed by several authors [5–8]. The second finding is that at relatively low temperature the amplitude of the $Z_1$ oscillation of the electronic stopping power of the plasma is even larger than that of the cold solids. Although we have not found any experimental group reporting this phenomenon, it can be understood as caused by the shell effects similarly to the $Z_1$ oscillation of the stopping powers in cold solids [13]. The difference between a cold solid and plasma is that the electronic stopping is determined by the scattering of the electrons at the Fermi level in the case of stopping in a cold solid, while the electrons in a plasma with all kinds of momentum will contribute to the stopping. The electron capture cross section, which is usually lower for free electrons than that for bound electrons, may depend more substantially on the structure of the incident ions in the case of a plasma target. Therefore, the effective charge of the projectile may be more sensitive to its own structure in the plasma targets at relatively low electron temperature. The third result we found is that the amplitude of the $Z_1$ oscillation in the plasma targets becomes smaller with increasing electron temperature. It is easy to understand that when the temperature of the electrons in the plasma is high enough the thermal movement of the electrons will eventually eliminate the effect of the shell structures of the projectiles on the stopping process and the $Z_1$ oscillation will thereby be eliminated.

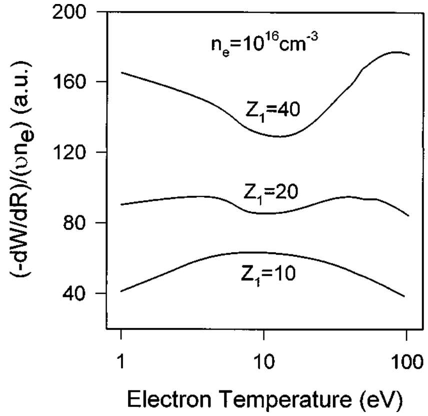


FIG. 2. Variation of the electronic stopping power of plasma targets for heavy ions with the plasma electron temperature.

Figure 2 shows the electronic stopping power of the hot plasma varying with the electron temperature. The stopping behaviors are different for different species of heavy ions. However, we can find a common feature: When the temperature of plasma exceeds a certain value, for example, 10 eV for $Z_1 = 10$, 50 eV for $Z_1 = 20$, and 90 eV for $Z_1 = 40$, the stopping power decreases gradually. This reduction of stopping power will cause the lengthening of the ion range in the plasma target and this extension of the ion range with increasing electron temperature is considered the physical origin of the so-called Tokamak wet-wood burners [22]. All of the results of this paper are just theoretical predictions and it would be nice to compare them with some experimental data. Unfortunately, the data we can find relating the stopping power of heavy ions in hot plasma all refer to heavy ions with very high kinetic energy (several MeV/nucleon) [2–4]. We hope that the data relating the slow heavy ions penetrating hot plasmas will emerge in the near future.

Summarizing, the electronic stopping powers of heavy ions at low velocity in plasma targets are calculated in the frame of quantum scattering theory. We find that the $Z_1$ oscillation of the electronic stopping power in a plasma target at relatively low temperature is even more evident than that in cold solids, but it becomes weaker with increasing electron temperature. This can be explained by the competition mechanism between the shell effects and the electron thermal movements. The extension of this method to finite projectile velocity regions is currently under way in our group.

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[1] R. C. Arnold and J. Meyer-ter-Vehn, Rep. Prog. Phys. **50**, 559 (1987).
[2] D. H. H. Hoffmann, K. Weyrich, and H. Wahl, Phys. Rev. A **42**, 2313 (1990).
[3] K. G. Dietrich, K. Mahrt-Olt, J. Jacoby, E. Boggasch, M. Winkler, B. Heimrich, and D. H. H. Hoffmann, Laser Part. Beams **8**, 583 (1990).
[4] J. Jacoby, D. H. H. Hoffmann, W. Laux, R. W. Müller, H. Wahl, K. Weyrich, E. Boggasch, B. Heimrich, C. Stöckl, H. Wetzler, and S. Miyamoto, Phys. Rev. Lett. **74**, 1550 (1995).
[5] E. Nardi and Z. Zinamon, Phys. Rev. Lett. **49**, 1251 (1982).
[6] Thomas A. Mehlhorn, J. Appl. Phys. **52**, 6522 (1981).
[7] Thomas Peter and Jürgen Meyer-ter-Vehn, Phys. Rev. A **43**, 1998 (1991).
[8] Thomas Peter and Jürgen Meyer-ter-Vehn, Phys. Rev. A **43**, 2015 (1991).
[9] Yong-Dae Jung, Phys. Rev. E **55**, 3369 (1997).
[10] D. O. Gericke, M. Schlanges, and W. D. Kraeft, Phys. Lett. A **222**, 241 (1996).
[11] P. Hvelplund and B. Fastrup, Phys. Rev. **165**, 408 (1968).
[12] J. H. Ormrod, J. R. Macdonald, and H. E. Duckworth, Can. J. Phys. **41**, 1424 (1963).
[13] P. M. Echenique, R. M. Nieminen, J. C. Ashley, and R. H. Ritchie, Phys. Rev. A **33**, 897 (1986).
[14] Yueyuan Xia, Chunyu Tan, and W. N. Lennard, Nucl. Instrum. Methods Phys. Res. B **90**, 88 (1994).
[15] W. B. Thompson, *An Introduction to Plasma Physics* (Pergamon, Oxford, 1962), p. 188.
[16] A. I. Akhiezer, *Plasma Electrodynamics* (Pergamon, Oxford, 1975), Vol. 2, p. 126.
[17] A. Sitenko and V. Malner, *Plasma Physics Theory* (Chapman & Hall, London, 1995), p. 141.
[18] E. Clementi and C. Roetti, At. Data Nucl. Data Tables **14**, 188 (1974).
[19] M. J. Puska and R. M. Nieminen, Phys. Rev. B **27**, 6121 (1983).
[20] Yuguang Mu, Yueyuan Xia, and Liangmo Mei, Phys. Rev. B **55**, 6188 (1997).
[21] T. L. Ferrell and R. H. Ritchie, Phys. Rev. B **16**, 115 (1977).
[22] J. M. Dawson, H. P. Furth, and F. H. Tenney, Phys. Rev. Lett. **26**, 1165 (1971).